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322311(14)

B. E. (Third Semester) Examination, 2020

APR-MAY

(Old Scheme)

(CSE, IT Engg. Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : *Part (a) of each unit is compulsory. All questions are required to be answered, selecting any **two** from (b), (c) and (d). Area under the normal curve table is allowed*

Unit-I

1. (a) Define periodic function with an example. 2

(b) Find the fourier series for the function $f(x)$, if $f(x) = x\cos x$ in $-\pi < x < \pi$. 7

(c) Obtain a half range cosine series for

$$f(x)=\begin{cases} kx, & 0\le x\le l/2 \\ k(l-x), & l/2\le x\le l \end{cases}$$ 7

(d) Find out the constant term and the co-efficient of the first sine and cosine terms in the fourier series of y as given in the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Unit-II

2. (a) Define unit step function and write their laplace transform. 2

(b) (i) Find the laplace transform of

$$\frac{e^{at}-\cos bt}{t}$$ 4

(ii) Evaluate :

$$\int_0^{\infty} e^{-2t}\cdot t\cos t\,dt$$ 3

(c) Prove that :

$$L^{-1}\left\{\frac{s}{s^4+s^2+1}\right\}=\frac{2}{\sqrt{3}}\ \sin h\cdot\sin\frac{\sqrt{3}}{2}t$$ 7

(d) Using the method of Laplace transform, solve :

$$\left(D^3-3D^2+3D-1\right)y=t^2e^t$$

given that $y(0)=1,\ y'(0)=0,\ y''(0)=-2$. 7

Unit-III

3. (a) State Cauchy's residue theorem. 2

(b) If $f(z)=u+iv$ is an analytic function and $z=re^{i\theta}$, where u, v, r, θ are all real, then show that the Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial V}{\partial \theta},\quad \frac{\partial V}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Also deduce that :

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$$ 7

(c) Evaluate

$$\int_C \frac{z^2-z+1}{z-1}dz$$

where C is the circle :

(i) $|z|=1$ and

(ii) $|z| = \frac{1}{2}$ 7

(d) Apply calculus of residues to prove that : 7

$$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta = \frac{\pi}{6}$$

Unit-IV

4. (a) Obtain the partial differential equation by eliminating the arbitrary function f from

$$z = f\left(\frac{y}{x}\right)$$ 2

(b) Solve : 7

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$

(c) Solve : 7

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \cos 2x \cdot \cos 3y$$

(d) Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

given that $u(x, 0) = 6e^{-3x}$. 7

Unit-V

5. (a) If a random variable has a Poisson distribution such that P (1) = P (2), then find mean of the distribution. 2

(b) The probability density $f(x)$ of a continuous random variable is given by 7

$$f(x) = y_0 x(2-x), \quad 0 \le x \le 2$$

Find the value of y_0, mean and variance of x.

(c) The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that :

(i) exactly two will be defective,

(ii) at least two will be defective and

(iii) none will be defective 7

(d) If X is a normal variate with mean 30 and standard deviation 5, then find the probabilities that : 7

(i) $26 \le X \le 40$,

(ii) $X \ge 45$ and

(iii) $|X - 30| > 5$